

The Golden Ratio, Phi and the Fibonacci Expansion...

Most people are familiar with the number (π) pi, 3.14..., since it is one of the most ubiquitous irrational numbers known to man. But, there is another irrational number that has the same propensity for popping up and is not as well known as pi. This wonderful number is (Φ) phi, and it has a tendency to turn up in a great number of places, in art, science and life, in general. Throughout history, the ratio for length to width of rectangles of 1.61803398874989484820 has been considered the most pleasing to the eye. This ratio was named the "golden ratio" by the Greeks. In the world of mathematics, the numeric value is called "phi", named for the Greek sculptor Phidias. The space between the columns form golden rectangles. There are golden rectangles throughout this structure which is found in Athens, Greece.

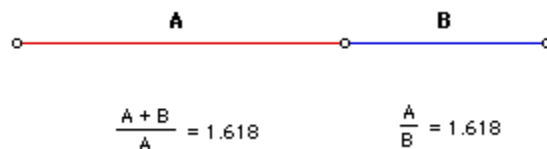
One way to find Phi is to consider the solutions to the equation

$$x^2 - x - 1 = 0$$

When solving this equation we find that the roots are:

$$x = \frac{1+\sqrt{5}}{2} \sim 1.618... \text{ or } x = \frac{1-\sqrt{5}}{2} \sim -.618...$$

This profound number, 1.618 has many uses in math and science. For example, phi may be found in many geometrical shapes, but instead of representing it as an irrational number, we can express it in the following way: Given that there is a line segment, we can divide it into two segments A and B, in such a way that the length of the entire segment is to the length of the segment A as the length of segment A is to the length of segment B. If we calculate these ratios, we see that we get an approximation of the Golden Ratio.



Another expression that represents and approximates phi is the Fibonacci expansion.

Fibonacci was a mathematician that approximated phi by the following equation:

If we take a number, starting with the value "1" and add it preceding value (starting with zero) then continue with subsequent values, we determine the following pattern:

1+1=2 ...

2+1=3 ... the sum (2) plus the previous value (1) equals the new sum (3).

3+2=5 ... The new sum (3) plus the previous value (2) equals (5)

3+5=8... 8+5=13... 13+8=21... 21+13=34... 34+21=55...

This series of sums, 1, 2, 3, 5, 8, 13, 21, 34, 55... are known as the Fibonacci series. If we take the ratio of two successive numbers in Fibonacci's series, (1, 2, 3, 5, 8, 13, ..) and we divide each by the number before it, we will find the following series of numbers:

$1/1 = 1$, $2/1 = 2$, $3/2 = 1.5$, $5/3 = 1.666\dots$, $8/5 = 1.6$, $13/8 = 1.625$, $21/13 = 1.61538\dots$

As this series progresses, the value approaches phi.

So what is the fuss about?

This profound number appears in nature extensively and the Fibonacci numbers play a significant role in nature and in art and architecture. For example, on a tree when a twig comes out from a branch, the next twig to come out from the branch will be slightly farther out on the branch and will be rotated about the branch at some angle. The next twig will come out farther along the branch, rotated at the same angle. Interestingly, the number of twigs needed for the last twig to come out of the branch at the same angle as the first twig is always a number that appears in the Fibonacci sequence, no matter what kind of tree it is. An apple tree might need 5 and a pear tree might need 8, but the number needed for any tree will always occur in the Fibonacci sequence.

Another example of the Fibonacci sequence can be seen in a Nautilus shell. Those are those shells that curl around on themselves. Inside are little chambers and you guessed it - let's say the first chamber has a volume of 1; then the second chamber has a volume of 2, the third chamber has a volume of 3, the fourth has a volume of 5, and so on: the Fibonacci sequence.

Two more examples: pineapples and turtles. One might ask what turtles and pineapples have in common, but the answer is of course the Fibonacci sequence (and their shells/rinds). A pineapple has shapes on its rind, bigger shapes on the bottom of it getting smaller as you go up. If you take one of the shapes on the bottom and go diagonally upward (to your right or left) you see that these shapes are basically the same only scaled to a smaller size as you go up. If we say the bottom has a size of 13, then we'll notice that the shape directly to the upper left (or right) has size 8. The shape to the upper left (or right) of this next shape has size 5, and so on down to one. The size you need to start with depends on the size of the pineapple and how many little shapes are in a sequence. On many plants, the number of

petals is a Fibonacci number: buttercups have 5 petals; lilies and iris have 3 petals; some delphiniums have 8; corn marigolds have 13 petals; some asters have 21 whereas daisies can be found with 34, 55 or even 89 petals.

For some turtles basically the same thing happens. Turtles have shells that have what one could call "tiles" on them. These tiles are the same shape but appear in different sizes. The sizes correspond to entries in the Fibonacci sequence.

Additionally, here are more examples of art and architecture which have employed the golden rectangle. This first example of the Great Pyramid of Giza is believed to be 4,600 years old, which was long before the Greeks. Its dimensions are also based on the Golden Ratio. Websites about the pyramid gives very extensive details on this.

Many artists who lived after Phidias have used this proportion. Leonardo Da Vinci called it the "divine proportion" and featured it in many of his paintings, including the famous "Mona Lisa". Da Vinci did an entire exploration of the human body and the ratios of the lengths of various body parts.